




Algebra 2 – Formula & Information Sheet (PARCC (NY) Version)

<p>Exponents: $x^0 = 1$ $x^{-m} = \frac{1}{x^m}$ $(xy)^n = x^n \cdot y^n$ $x^m \cdot x^n = x^{m+n}$ $(x^n)^m = x^{n \cdot m}$ $\frac{x^m}{x^n} = x^{m-n}$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$</p>	<p>Complex Numbers: $\sqrt{-1} = i$ $i^2 = -1$ $\sqrt{-a} = i\sqrt{a}; a \geq 0$ $i^4 = i^2 = -1$ Divide exponent by 4, use remainder, solve. (a + bi) conjugate (a - bi) (a + bi)(a - bi) = a² + b²</p>	<p>Logarithms $y = \log_b x \Leftrightarrow x = b^y$ $\ln x = \log_e x$ (natural log) $e = 2.71828\dots$ $\log x = \log_{10} x$ (common log) Change of base formula: $\log_b a = \frac{\log a}{\log b}$ $\log_b(m^r) = r \log_b m$ Domain: $\log_b x$ is $x > 0$</p>	<p>Properties of Logs: $\log_b b = 1$ $\log_b 1 = 0$ $\log_b(m \cdot n) = \log_b m + \log_b n$ $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$</p>
<p>Radicals: Use fractional exponents. $\sqrt[n]{x} = x^{\frac{1}{n}}$ $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$ $\sqrt[n]{a^n} = a$ $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ Simplify: look for perfect powers. $\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$ $\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \cdot 9x^9y^6z^3} = 2x^3y^2z\sqrt[3]{9y^2}$ Equations: isolate radical; square both sides to eliminate radical; combine; solve. $2x - 5\sqrt{x} - 3 = 0 \rightarrow (2x - 3)^2 = (5\sqrt{x})^2$ $4x^2 - 12x + 9 = 25x \rightarrow \text{solve: } x = 9; x = 1/4$ Check Answers. Answer only $x = 9$.</p>	<p>Factoring: Look for GCF (greatest common factor) $ab + ac = a(b + c)$ $x^2 - a^2 = (x - a)(x + a)$ Perfect Squares: $(x + a)^2 = x^2 + 2ax + a^2$ $(x - a)^2 = x^2 - 2ax + a^2$ Factor by Grouping: $x^3 + 2x^2 - 3x - 6 = (x^3 + 2x^2) - (3x + 6)$ $= x^2(x + 2) - 3(x + 2) = (x^2 + 3)(x + 2)$</p>	<p>Sequences Arithmetic: $a_n = a_1 + (n - 1)d$ $S_n = \frac{n(a_1 + a_n)}{2}$ Geometric: $a_n = a_1 \cdot r^{n-1}$ $S_n = \frac{a_1(1 - r^n)}{1 - r}$ or $\frac{a_1 - a_1r^n}{1 - r}; r \neq 1$ Recursive: Example: $a_1 = 4; a_n = 2a_{n-1}$</p>	<p>Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.) Solve by factoring, completing the square, quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $b^2 - 4ac > 0$ two real unequal roots $b^2 - 4ac = 0$ repeated real roots $b^2 - 4ac < 0$ two complex roots Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$ Completing the square: $x^2 - 2x - 5 = 0$ 1. If other than one, divide by coefficient of x^2 2. Move constant term to other side $x^2 - 2x = 5$ 3. Take half of coefficient of x, square it, add to both sides $x^2 - 2x + 1 = 5 + 1$ 4. Factor perfect square on left side. $(x - 1)^2 = 6$ 5. Use square root property to solve and get two answers. $x = 1 \pm \sqrt{6}$</p>
<p>Polynomials: $(x^2 + x^2 + 2)(x + 3)$ Long division: $\begin{array}{r} x^2 - 2x + 6 \\ x + 3 \overline{) x^3 + x^2 + 0x + 2} \\ \underline{x^3 + 3x^2} \\ -2x^2 + 0x \\ \underline{-2x^2 - 6x} \\ 6x + 2 \\ \underline{6x + 18} \\ -16 \end{array}$ Remainder Th^m: $f(x) = x^3 + x^2 + 2$ $f(-3) = \text{remainder} = -16$</p>	<p>Inequalities: $x^2 + x - 12 \leq 0$ Change to =, factor, locate critical points on number line, check each section. $(x + 4)(x - 3) = 0$ $x = -4; x = 3$</p> <p>ANSWER: $-4 \leq x \leq 3$ or $[-4, 3]$ (in interval notation)</p>		

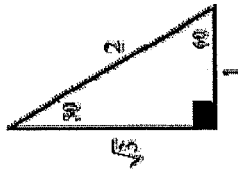
<p>Exponentials $e^x = \exp(x)$ $b^x = b^y \rightarrow x = y$ ($b > 0$ and $b \neq 1$) If the bases are the same, set the exponents equal and solve. Solving exponential equations: 1. Isolate exponential expression. 2. Take \log or \ln of both sides. 3. Solve for the variable.</p> <p>$\ln(x)$ and e^x are inverse functions $\ln e^x = x$ $e^{\ln x} = x$ $\ln e = 1$ $e^{\ln 4} = 4$ $e^{2 \ln 3} = e^{\ln 3^2} = 9$</p> <p>Functions: A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.</p> <p>Vertical Line Test: Is this graph a function? Horizontal line test: Will inverse be a function? Domain: x-values used; Range: y-values used Odd: $f(-x) = -f(x)$ Even: $f(-x) = f(x)$ 1-to-1: no y-element used more than once. Composition: $(f \circ g)(x) = f(g(x))$ Inverse functions f & g: $f(g(x)) = g(f(x)) = x$</p> <p>Transformations:</p> <ul style="list-style-type: none"> • $-f(x)$ over x-axis; • $f(-x)$ over y-axis • horizontal shift: $f(x+h)$ left; $f(x-h)$ right • vertical shift: $f(x)+k$ up; $f(x)-k$ down • $f(ax)$ horizontal compress ($a > 1$); stretch ($0 < a < 1$) • $af(x)$ vertical stretch ($a > 1$); shrink ($0 < a < 1$) 	<p>Working with Rationals (Fractions): Simplify: remember to look for a factoring of -1: $\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$</p> <p>Add: Get the common denominator. Factor first if possible: Multiply and Divide: Factor First</p> <p>Rational Inequalities $\frac{x^2 - 2x - 15}{x - 2} \geq 0$ The critical values from factoring the numerator are -3, 5. The denominator is zero at $x = 2$. Place on number line, and test sections. </p>	<p>Solving Rational Equations: Get rid of the denominators by mult. all terms by common denominator. $\frac{22}{2x^2 - 9x - 5} - \frac{3}{2x + 1} = \frac{2}{x - 5}$ <i>multiply all by $2x^2 - 9x - 5$ and get</i> $22 - 3(x - 5) = 2(2x + 1)$ $22 - 3x + 15 = 4x + 2$ $37 - 3x = 4x + 2$ $35 = 7x$ $5 = x$ Great! But the only problem is that $x = 5$ does not CHECK!!! There is no solution. Extraneous root. Motto: Always CHECK ANSWERS.</p>
	<p>Exponential functions: $y = a \cdot b^x$ Growth and Decay: $y = y_0 e^{kt}$ $A = A_0 e^{k(t-t_0)} + B_0$ (PARCC) Finance: $A = P \left(1 + \frac{r}{n} \right)^{nt}$ regular compound P principal, r annual interest rate, n times per yr., t years, A amount after time t weekly $n = 52$ monthly $n = 12$ yearly $n = 1$ quarterly $n = 4$ $A = Pe^{rt}$ compound continuously</p>	<p>Equations of Circles: $x^2 + y^2 = r^2$ center origin radius r $(x-h)^2 + (y-k)^2 = r^2$ center at (h,k) $x^2 + y^2 + Cx + Dy + E = 0$ general form</p> <p>Complex Fractions: Remember that the fraction bar means divide: Method 1: Get common denominator top and bottom $\frac{.2}{\frac{x^2}{4}} - \frac{4}{\frac{2-4x}{x^2}} = \frac{2-4x}{x^2} \div \frac{2-4x}{x^2} = \frac{2-4x}{x^2} \cdot \frac{x^2}{2-4x} = -1$</p> <p>Method 2: Mult. all terms by common denominator for all. $\frac{2}{x^2} - \frac{4}{x} = \frac{2-4x}{x^2} \cdot \frac{x}{x} = \frac{2-4x}{4x-2} = -1$</p>

Trigonometry

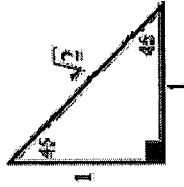
Radians and Degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees; } 1 \text{ degree} = \frac{\pi}{180} \text{ radians}$$

Special Right Triangles



30°-60°-90° triangle
side opposite 30° = ½ hypotenuse
side opposite 60° = ½ hypotenuse $\sqrt{3}$



45°-45°-90° triangle
hypotenuse = leg $\sqrt{2}$
leg = ½ hypotenuse $\sqrt{2}$

Degrees, Minutes, Seconds:

1 degree = 60 minutes

1 minute = 60 seconds

46° 34' 24"

Arc Length of a Circle = θr (in radians)

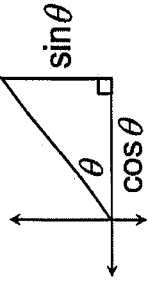
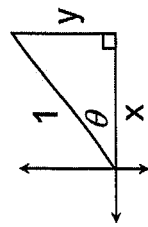
$$\theta = \frac{s}{r} \text{ or } s = r\theta$$

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Unit Circle Relationships:



Add / Subtract Angles:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$\cos(2A) = 2 \cos^2 A - 1$$

$$\cos(2A) = 1 - 2 \sin^2 A$$

Reference triangles
are drawn to the x-axis.



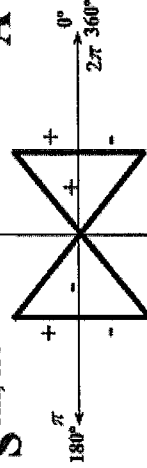
Make A BowTie

Quadrant II

Positive:
S
sin, csc

Quadrant I

Positive:
ALL A



Positive:
T
tan, cot

Positive:
C
cos, sec

Quadrant III

Quadrant IV

Inverse notation:

$$\arcsin(x) = \sin^{-1}(x)$$

$$\arctan(x) = \tan^{-1}(x)$$

$$\arccos(x) = \cos^{-1}(x)$$

$$\text{arccsc}(x) = \csc^{-1}(x)$$

Trig Functions

$$\sin \theta = \frac{o}{h}; \quad \cos \theta = \frac{a}{h}; \quad \tan \theta = \frac{o}{a}$$

$$\csc \theta = \frac{h}{o}; \quad \sec \theta = \frac{h}{a}; \quad \cot \theta = \frac{a}{o}$$

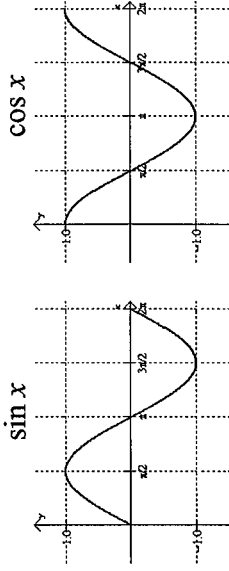
Reciprocal Functions

$$\sin \theta = \frac{1}{\csc \theta}; \quad \cos \theta = \frac{1}{\sec \theta}; \quad \tan \theta = \frac{1}{\cot \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}; \quad \sec \theta = \frac{1}{\cos \theta}; \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Trig Graphs



sinusoidal curve = any curve expressed as
 $y = A \sin(B(x - C)) + D$

amplitude (A) = $\frac{1}{2} |\text{max} - \text{min}|$ (think height)

period = horizontal length of 1 complete cycle

frequency (B) = number of cycles in 2π (period)

horizontal shift (C) – movement left/right

vertical shift (D) – movement up/down

midline – horizontal line between the max and min y-values (through the middle y-value).

Statistics and Probability

Statistics:

$$\text{mean} = \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

median = middle number in ordered data

mode = value occurring most often

range = difference between largest and smallest

standard deviation (S.D.):

$$\text{population S.D.} = \sigma_x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

S_x = sample standard deviation

σ_x = population standard deviation

Calculator: Given mean μ and pop. σ ,

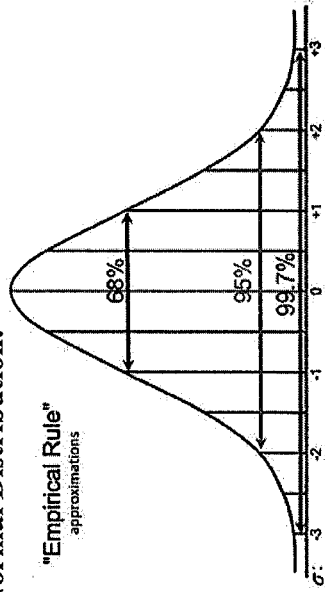
use normalcdf - probability between values

use normalpdf - graph normal curve, area

Z-Scores: A measure of position that indicates the number of S.D. from mean. Uses area under normal curve. Must know mean μ and pop. σ .

$$z = \frac{x - \mu}{\sigma} \text{ and use a Z-chart.}$$

Normal Distribution:



68% - 95% - 99.7% refers 1,2,3 S.D. from mean

Confidence Intervals and Levels:

Levels: usually 95%

Proportions:

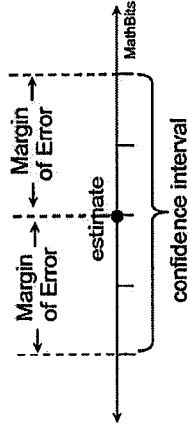
Standard Error: S.D. of sampling distribution

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}; p = \text{mean of sampling}$$

Margin of Error: Standard Error x critical value of confidence level (95% = 2)

$$MOE = 2 \sqrt{\frac{p(1-p)}{n}}$$

Confidence Interval:



Confidence Interval: estimate \pm MOE

Means:

Standard Error: S.D. of sampling distribution

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}; \sigma = \text{population S.D., } n = \text{sample \#}$$

$$\text{Margin of Error: } MOE = 2 \frac{\sigma}{\sqrt{n}}$$

Probability and Two-Way Frequency Table:

ACTIVITIES	Football	Not Football	TOTAL
Drama Club	35	30	65
Not Drama Club	60	42	102
TOTAL	95	72	167

$$\bullet P(\text{Football and Drama}) = 35/167$$

$$\bullet P(\text{Football} | \text{Drama}) = 35/65$$

$$\bullet P(\text{Not Drama}) = 102/167$$

$$\bullet P(\text{Drama} | \text{Not Football}) = 30/72$$

Probability

Empirical Probability

$$P(E) = \frac{\# \text{ of times event } E \text{ occurs}}{\text{total \# of observed occurrences}}$$

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\# \text{ of outcomes in } E}{\text{total \# of outcomes in } S}$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

for independent events

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

for dependent events

$$P(A^C) = 1 - P(A) \text{ complement}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

for not mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$

for mutually exclusive

Conditional Probability ("given")

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

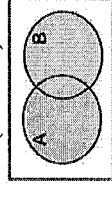
for dependent events

$$P(B|A) = P(B) \text{ or } P(A|B) = P(A)$$

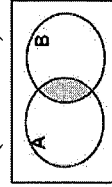
for independent events

Venn Diagrams:

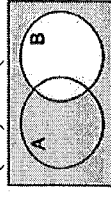
P(A or B)



P(A and B)



P(B^C) = P(not B)



P(A and B^C)

