Algebra 2 – Formula & Information Sheet (PARCC (NY) Version)

Exponents: $x^0 = 1$

$x^{-m} = \frac{1}{x^m}$ $\quad (xy)^n = x^n \bullet y^n$

$x^m \bullet x^n = x^{m+n}$ $\quad (x^n)^m = x^{n \bullet m}$

$\frac{x^m}{x^n} = x^{m-n}$ $\quad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Radicals: Use fractional exponents.

$\sqrt[a]{x} = x^{\frac{1}{a}}$ $\quad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{a^n} = a$ $\quad \sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ $\quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: look for perfect powers.

$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$

$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \bullet 9x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2}$

Equations: isolate radical; square both sides to eliminate radical; combine; solve.

$2x - 5\sqrt{x} - 3 = 0 \quad \rightarrow \quad (2x-3)^2 = (5\sqrt{x})^2$

$4x^2 - 12x + 9 = 25x \rightarrow solve: x = 9; x = 1/4$

Check Answers. Answer only $x = 9$.

Polynomials: $(x^3 + x^2 + 2)/(x + 3)$

Long division:

$$\begin{array}{r} x^2 - 2x + 6 \\ x+3 \overline{\smash{)}\, x^3 + x^2 + 0x + 2} \\ \underline{x^3 + 3x^2} \\ -2x^2 + 0x \\ \underline{-2x^2 - 6x} \\ 6x + 2 \\ \underline{6x + 18} \\ -16 \end{array}$$

Synthetic division:

$$\begin{array}{r|rrrr} -3 & 1 & 1 & 0 & 2 \\ & & -3 & 6 & -18 \\ \hline & 1 & -2 & 6 & -16 \end{array}$$

Remainder Th^m:

$f(x) = x^3 + x^2 + 2$

$f(-3)$ = remainder = -16

Complex Numbers:

$\sqrt{-1} = i$ $\quad \sqrt{-a} = i\sqrt{a};\ a \geq 0$

$i^2 = -1$ $\quad i^{14} = i^2 = -1$ Divide exponent by 4, use remainder, solve.

$(a + bi)$ conjugate $(a - bi)$

$(a + bi)(a - bi) = a^2 + b^2$

Factoring:

Look for GCF (greatest common factor)

$ab + ac = a(b + c)$

$x^2 - a^2 = (x - a)(x + a)$

Perfect Squares:

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

Sum/Diff. Cubes:

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Factor by Grouping:

$x^3 + 2x^2 - 3x - 6 = (x^3 + 2x^2) - (3x + 6)$

$= x^2(x + 2) - 3(x + 2) = (x^2 + 3)(x + 2)$

Sequences

Arithmetic: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Geometric: $a_n = a_1 \bullet r^{n-1}$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} \text{ or } \frac{a_1 - a_1r^n}{1 - r}; r \neq 1$$

Recursive: Example:

$a_1 = 4; \quad a_n = 2a_{n-1}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ (natural log)

$e = 2.71828\ldots$

$\log x = \log_{10} x$ (common log)

Change of base formula:

$\log_b a = \frac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\quad \log_b 1 = 0$

$\log_b(m \bullet n) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^r) = r\log_b m$

Domain: $\log_b x$ is $x > 0$

Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.)

Solve by factoring, completing the square, quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac > 0$ two real unequal roots

$b^2 - 4ac = 0$ repeated real roots

$b^2 - 4ac < 0$ two complex roots

Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$

Completing the square: $x^2 - 2x - 5 = 0$

1. If other than one, divide by coefficient of x^2
2. Move constant term to other side $\quad x^2 - 2x = 5$
3. Take half of coefficient of x, square it, add to both sides $\quad x^2 - 2x + \boxed{1} = 5 + \boxed{1}$
4. Factor perfect square on left side. $\quad (x-1)^2 = 6$
5. Use square root property to solve and get two answers. $\quad x = 1 \pm \sqrt{6}$

Inequalities: $x^2 + x - 12 \leq 0$ Change to =, factor, locate critical points on number line, check each section.

$(x + 4)(x - 3) = 0$

$x = -4; \ x = 3$

ANSWER: $-4 \leq x \leq 3$ or $[-4, 3]$ (in interval notation)

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Exponentials $e^x = \exp(x)$

$b^x = b^y \rightarrow x = y \quad (b > 0 \text{ and } b \neq 1)$

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x \qquad e^{\ln x} = x$

$\ln e = 1 \qquad e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

Functions: A function is a set of ordered pairs in which each *x*-element has only ONE *y*-element associated with it.

Vertical Line Test: Is this graph a function?
Horizontal line test: Will inverse be a function?
Domain: *x*-values used; **Range:** *y*-values used
Odd: $f(-x) = -f(x)$ **Even:** $f(-x) = f(x)$
1-to-1: no *y*-element used more than once.
Composition: $(f \circ g)(x) = f(g(x))$
Inverse functions *f* & *g*: $f(g(x)) = g(f(x)) = x$

Transformations:

- $-f(x)$ over *x*-axis;
- $f(-x)$ over y-axis
- horizontal shift: $f(x+h)$ left; $f(x-h)$ right
- vertical shift: $f(x)+k$ up; $f(x)-k$ down
- $f(ax)$ horizontal compress ($a > 1$); stretch ($0 < a < 1$)
- $af(x)$ vertical stretch ($a > 1$); shrink ($0 < a < 1$)

Working with Rationals (Fractions):
Simplify:
remember to look for a factoring of -1:

$$\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$$

Add: Get the common denominator.
Factor first if possible:
Multiply and Divide: Factor First

Rational Inequalities

$$\frac{x^2-2x-15}{x-2} \geq 0$$ The critical values from factoring the numerator are -3, 5.
The denominator is zero at $x = 2$.
Place on number line, and test sections.

Exponential functions:

$y = a \cdot b^x$

Growth and Decay: $y = y_0 e^{kt}$

$A = A_0 e^{k(t-t_0)} + B_0$ (PARCC)

Finance:

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$ regular compound

P principal, *r* annual interest rate, *n* times per yr., *t* years, *A* amount after time *t*

weekly $n = 52$
monthly $n = 12$
yearly $n = 1$
quarterly $n = 4$

$A = Pe^{r \cdot t}$ compound continuously

Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ *and get*

$22 - 3(x-5) = 2(2x+1)$

$22 - 3x + 15 = 4x + 2$

$37 - 3x = 4x + 2$

$35 = 7x$

$5 = x$

Great! But the only problem is that *x* = 5 does not CHECK!!!! There is no solution.
Extraneous root.
Motto: Always CHECK ANSWERS.

Equations of Circles: $x^2 + y^2 = r^2$ center origin

$(x-h)^2 + (y-k)^2 = r^2$ center at (*h*,*k*) radius *r*

$x^2 + y^2 + Cx + Dy + E = 0$ general form

Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{\overset{-1}{\cancel{2-4x}}}{\cancel{x^2}} \bullet \frac{\cancel{x^2}}{\cancel{4x-2}} = -1$$

Method 2: Mult. all terms by common denominator for all.

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{x^2 \bullet \frac{2}{x^2} - x^2 \bullet \frac{4}{x}}{x^2 \bullet \frac{4}{x} - x^2 \bullet \frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

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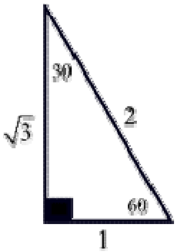
Trigonometry

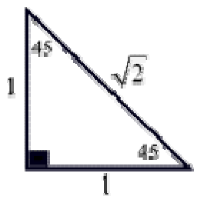
Radians and Degrees

1 radian = $\frac{180}{\pi}$ degrees; 1 degree = $\frac{\pi}{180}$ radians

Special Right Triangles



30º-60º-90º triangle
side opposite 30º = ½ hypotenuse
side opposite 60º = ½ hypotenuse $\sqrt{3}$



45º-45º-90º triangle
hypotenuse = leg $\sqrt{2}$
leg = ½ hypotenuse $\sqrt{2}$

Degrees, Minutes, Seconds:

1 degree = 60 minutes
1 minute = 60 seconds
46º 34' 24"

Arc Length of a Circle = θr (in radians)

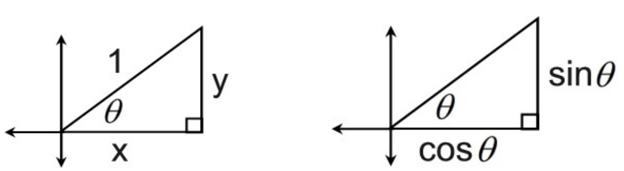
$$\theta = \frac{s}{r} \quad \text{or} \quad s = r\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

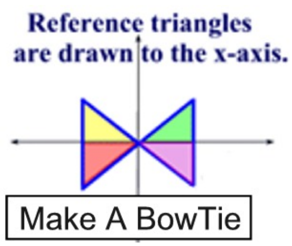
$$1 + \cot^2\theta = \csc^2\theta$$

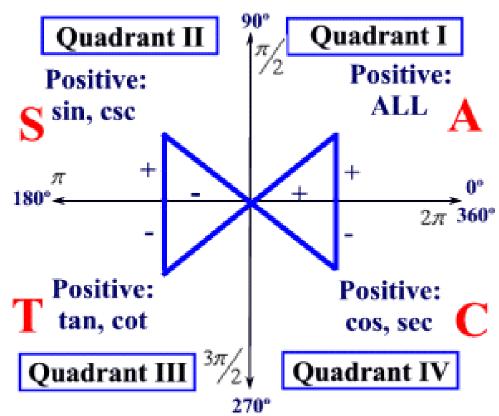
Unit Circle Relationships:



Add / Subtract Angles:

- sin(A + B) = sinAcosB + cosAsinB
- cos(A + B) = cosAcosB – sinAsinB
- sin(A – B) = sinAcosB – cosAsinB
- cos(A – B) = cosAcosB + sinAsinB
- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- sin(2A) = 2sinAcosA
- cos(2A) = $\cos^2$A – $\sin^2$A
- cos(2A) = 2$\cos^2$A – 1
- cos(2A) = 1 – 2$\sin^2$A





Inverse notation:

arcsin(x) = $\sin^{-1}(x)$ arccos(x) = $\cos^{-1}(x)$
arctan(x) = $\tan^{-1}(x)$

Trig Functions

$$\sin\theta = \frac{o}{h}; \quad \cos\theta = \frac{a}{h}; \quad \tan\theta = \frac{o}{a}$$

$$\csc\theta = \frac{h}{o}; \quad \sec\theta = \frac{h}{a}; \quad \cot\theta = \frac{a}{o}$$

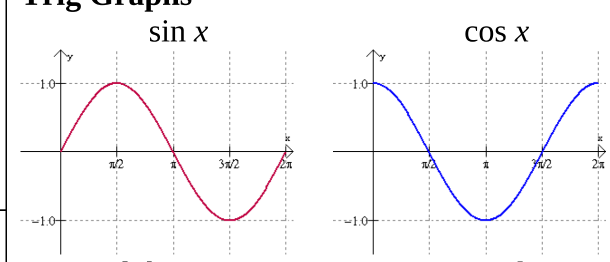
Reciprocal Functions

$$\sin\theta = \frac{1}{\csc\theta}; \quad \cos\theta = \frac{1}{\sec\theta}; \quad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}; \quad \sec\theta = \frac{1}{\cos\theta}; \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trig Graphs



sinusoidal curve = any curve expressed as
$y = A\sin(B(x - C)) + D$

amplitude (A) = ½ | max – min| *(think height)*

period = horizontal length of 1 complete cycle

frequency (B) = number of cycles in 2π (period)

horizontal shift (C) – movement left/right

vertical shift (D) – movement up/down

midline – horizontal line between the max and min y-values (through the middle y-value).

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Statistics and Probability

Statistics:

$$mean = \bar{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

median = middle number in ordered data
mode = value occurring most often
range = difference between largest and smallest

standard deviation (S.D.):

$$\text{population S.D: } \sigma x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

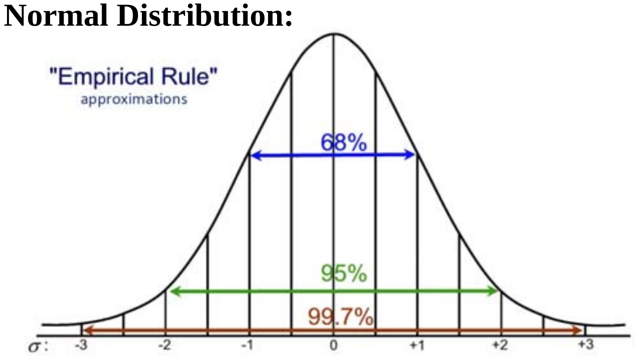
Sx = sample standard deviation
σ_x = population standard deviation

Calculator: Given mean μ and pop. σ,
use normalcdf - probability between values
use normalpdf – graph normal curve, area

Z-Scores: A measure of position that indicates the number of S.D. from mean. Uses area under normal curve. Must know mean μ and pop. σ.

$$z = \frac{x - \mu}{\sigma}$$ and use a Z-chart.

Normal Distribution:



68% - 95% - 99.7% refers 1,2,3 S.D. from mean

Confidence Intervals and Levels:

Levels: usually 95%

Proportions:

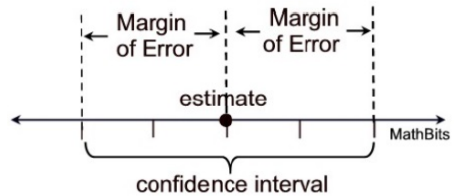
Standard Error: S.D. of sampling distribution

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$; p = mean of sampling

Margin of Error: Standard Error x critical value of confidence level (95% = 2)

$$MOE = 2\sqrt{\frac{p(1-p)}{n}}$$

Confidence Interval:



Confidence Interval: estimate ± MOE

Means:

Standard Error: S.D. of sampling distribution

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$; σ = population S.D., n = sample #

Margin of Error: $MOE = 2\frac{\sigma}{\sqrt{n}}$

Probability and Two-Way Frequency Table:

ACTIVITIES	Football	Not Football	TOTAL
Drama Club	35	30	65
Not Drama Club	60	42	102
TOTAL	95	72	167

- P(Football and Drama) = 35/167
- P(Football | Drama) = 35/65
- P(Not Drama) = 102/167
- P(Drama | Not Football) = 30/72

Probability

Empirical Probability

$$P(E) = \frac{\text{\# of times event } E \text{ occurs}}{\text{total \# of observed occurrences}}$$

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{\# of outcomes in } E}{\text{total \# of outcomes in } S}$$

P(A and B) = P(A)•P(B)
for independent events
P(A and B) = P(A)•P(B| A)
for dependent events

$P(A^C) = 1 - P(A)$ complement

P(A or B) = P(A) + P(B) – P(A and B)
for not mutually exclusive
P(A or B) = P(A) + P(B)
for mutually exclusive

Conditional Probability ("given")

$$P(B\,|\,A) = \frac{P(A \text{ and } B)}{P(A)}$$

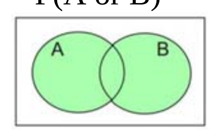
for dependent events

$P(B\,|\,A) = P(B)$ *or* $P(A\,|\,B) = P(A)$

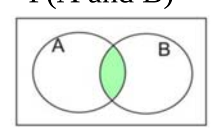
for independent events

Venn Diagrams:

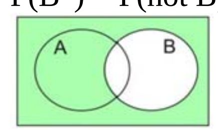
P(A or B)



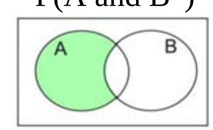
P(A and B)



$P(B^C)$ = P(not B)



P(A and B^C)



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